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An Incentive-Based Multistage Expansion Planning Model for Smart Distribution Systems

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***Abstract*—The deployment of smart grids has facilitated the integration of a variety of investor assets into power distribution systems, giving rise to the consequent necessity for positive and active interaction between those investors and local distribution companies (LDCs). This paper proposes a novel incentive-based distribution system expansion planning (IDSEP) model that enables an LDC and distributed generation (DG) investors to work in a collaborative way for their mutual benefit. Using the proposed model, the LDC would establish a bus-wise incentive program (BWIP) based on long-term contracts, which would encourage DG investors to integrate their projects at specific system buses that would benefit both parties. The model guarantees that the LDC will incur minimum expansion and operation costs while concurrently ensuring the feasibility of DG investors' projects. To derive appropriate incentives for each project, the model enforces several economic metrics including internal rate of return, profit investment ratio, and discounted payback period. All investment plans committed to by the LDC and the DG investors for the full extent of the planning period are then coordinated accordingly. Several linearization approaches are applied to convert the proposed model into an MILP model. The intermittent nature of both system demand and wind- and PV-based DG output power is handled probabilistically, and a number of DG technologies are taken into account. Case study results have demonstrated the value of the proposed model.**

***Index Terms*—Distributed generation planning, distribution system expansion planning, DG uncertainty modeling, incentives.**

NOMENCLATURE

A. Indices

i, j	Indices for system buses.
t	Index for time stages.
e	Index for uncertainty scenarios.
ij	Index for system branches.
u, c	Indices for substation alternatives.
a	Index for feeder alternatives.
dg	Index for DG types.
y	Index for the blocks in piecewise linearization.

B. Sets

Ω_N	Set of system buses.
Ω_{ES}, Ω_{CS}	Sets of existing and candidate substation buses.
Ω_{SS}	Set of all substations where $\Omega_{SS} = \Omega_{ES} \cup \Omega_{CS}$.
Ω_{EL}, Ω_{CL}	Sets of existing and candidate feeder branches.
Ω_L	Set of all branches where $\Omega_L = \Omega_{EL} \cup \Omega_{CL}$.
Ω_U	Set of alternatives for upgrading existing substations.
Ω_C	Set of alternatives for constructing new candidate substations.
Ω_{se}	Set of scenarios.
Ω_{DG}	Set of DG types. $\Omega_{DG} = \{CDG, WDG, PVDG\}$, where CDG is controllable DG, WDG is wind-based DG, and $PVDG$ is PV-based DG.
T	Set of time stages.

C. Parameters

C_u^{US}, C_c^{NS}	Costs of upgrading an existing substation and constructing a new candidate substation (US$).
C_a^{UF}, C_a^{NF}	Costs of upgrading an existing feeder and constructing a new candidate feeder (US$/km).
L_{ij}	Length of feeder *ij* (km).
α_e	Probability of scenario e.
φ	Total hours in one year ($\varphi = 8760$).
ω	Substation operation cost.
ε	Cost of energy losses (US$/MWh).
$C_{e,t}^E$	Market energy purchasing cost (US$/MWh).
$OP_{dg,e}$	Representative DG state output power as a percentage of the DG rated capacity for type dg in scenario e.
ρ_{dg}	Binary parameter (1 if a DG of type dg is considered; 0 otherwise).
τ	Interest rate.
K	Number of years in each stage.
DL_e	Representative load state as a percentage of the peak load in scenario e.
$P_{D_{i,t}}$	Nodal active power demand (MW).
$Q_{D_{i,t}}$	Nodal reactive power demand (MVAR).
G_{ij}, B_{ij}	Conductance and susceptance of branch *ij*.
R_{ij}, X_{ij}	Resistance and reactance of branch *ij*.
$\overline{S_{G_i}^{max}}$	Maximum existing substation capacity (MVA).
S_u^{US}	Existing substation upgrade capacity for alternative *u* (MVA).
S_c^{NS}	New substation capacity for alternative *c* (MVA).
$\overline{S_{ij}^{max}}$	Maximum existing feeder capacity (A).
S_a^P	New feeder capacity for alternative *a* (A).
$\underline{V}^{Min}, \overline{V}^{Max}$	Minimum and maximum voltage magnitude.
$\underline{\Delta V_i}^{Min}, \overline{\Delta V_i}^{Max}$	Minimum and maximum voltage magnitude deviation.
N_b	Total number of buses.
N_{ES}	Total number of existing substations.
C_{dg}^{IDG}	DG investment cost for type dg (US$/MW).
C_{dg}^{ODG}	DG operation cost for type dg (US$/MWh).
$IRR_{dg,i}$	Internal rate of return for a DG investor.
$\underline{\gamma}, \overline{\gamma}$	Minimum and maximum incentive prices (US$/MWh).
$\overline{DG_i}$	Maximum DG capacity at bus *i* (MW).
μ	DG penetration level, as a percentage.
M	Disjunctive factor, a large positive number.

$\bar{\Delta}^G$	Upper limit for each linear segment of $\Delta P_{G_{i,e,t,y}}$ and $\Delta Q_{G_{i,e,t,y}}$.
$\bar{\Delta}^L$	Upper limit for each linear segment of $\Delta P_{ij,e,t,y}$ and $\Delta Q_{ij,e,t,y}$.
Y	Number of blocks in piecewise linearization.

D. Variables

$\sigma_{i,u,t}$	Binary variable associated with upgrading an existing substation.
$u_{j,c,t}$	Binary variable associated with constructing a new substation.
$\beta_{ij,a,t}$	Binary variable associated with upgrading an existing feeder.
$z_{ij,a,t}$	Binary variable associated with constructing a new feeder.
$x_{ij,t}$	Binary variable associated with a feeder configuration (1 if feeder is ON; 0 otherwise).
$v_{h,dg,i}$	Binary variable associated with the binary expansion used for BWIP.
$S^{sqr}_{G_{i,e,t}}$	Square of the apparent power supplied by a substation.
$P_{G\,i,e,t}$	Active power supplied by a substation (MW).
$Q_{G\,i,e,t}$	Reactive power supplied by a substation (MVAR).
$\Delta P_{G_{i,e,t,y}}$	yth linear block of active substation power.
$\Delta Q_{G_{i,e,t,y}}$	yth linear block of reactive substation power.
$S^{sqr}_{ij,e,t}$	Square of the apparent power flow in the feeder.
$P_{ij,e,t}$	Active power flow in the feeder (MW).
$Q_{ij,e,t}$	Reactive power flow in the feeder (MVAR).
$\Delta P_{ij,e,t,y}$	yth linear block of active power flow in the feeder.
$\Delta Q_{ij,e,t,y}$	yth linear block of reactive power flow in the feeder.
$P^+_{ij,e,t}, P^-_{ij,e,t}$	Nonnegative variables used to replace $P_{ij,e,t}$.
$Q^+_{ij,e,t}, Q^-_{ij,e,t}$	Nonnegative variables used to replace $Q_{ij,e,t}$.
$Pl_{ij,e,t}$	Loss of active power by the feeder (MW).
$Ql_{ij,e,t}$	Loss of reactive power by the feeder (MVAR).
$P_{DG\,dg,i,t}$	DG rated capacity (MW).
$\gamma_{dg,i}$	DG BWIP incentive price (US$/MWh).
$d_{h,dg,i}$	Positive variable used for incentive price linearization.
$INC_{dg,i,t}$	Incentive cost for the DG (US$).
$V_{i,e,t}$	Bus voltage magnitude.
$\Delta V_{i,e,t}$	Bus voltage magnitude deviation from the nominal voltage.
$\delta_{i,e,t}$	Bus voltage angle.
$INVC^{DG}_{dg,i,t}$	DG investment cost.
$OPC^{DG}_{dg,i,e,t}$	DG operation cost.
$BEN^{DG}_{dg,i,e,t}$	Benefit from the sale of DG-generated energy.

I. Introduction

EVER-EXPANDING population growth and industrial market competition have been accompanied by a simultaneous increase in power consumption and electrical energy demand. Distribution system companies are solely responsible for meeting any anticipated increases in demand, making expansion plans for distribution system assets an essential top priority for planning engineers [1]. The bottom line is that the high costs of the vast investments involved in distribution networks dictate very careful planning and operation. Such tasks necessitate comprehensive economic planning tools that can select a feasible solution from a variety of available alternatives and resources in order to ensure reliable, affordable, and sustainable power delivery to customers. Moreover, there is a demand for planning models that can respond to independent private investments in power generation and distribution systems under the deregulation frameworks [2].

Distributed generation (DG) units are expected to play a pivotal role in addressing problems associated with distribution system expansion planning (DSEP) as well as to provide numerous technical and environmental advantages. However, a look at current distribution utility practices reveals that most LDCs are unwilling to invest in DG technologies because of two primary obstacles. First, distribution utilities, which are in fact struggling to survive in the competitive electricity market [3], have been subject to massive cost-cutting measures that have drastically reduced their capital budgets [4]. This shortage of funds plus the high initial costs of DGs deter LDCs from investing in these units [5]. Second, from a regulatory perspective, in many countries an unbundling rule for electricity market participants requires LDCs to be legally separate from generation facilities, thus in effect preventing LDCs from owning DGs [6], [7]. The result is that, in the majority of cases and as a dominant practice, DGs are owned and operated by private investors. The ultimate goal of these parties is to capture all of the benefits of the business, regardless of whether the locations of their projects are beneficial for the grid, for example with respect to deferring upgrading decisions or reducing losses. The key question is therefore how distribution utilities can take advantage of such DG projects and direct their integration to specific locations that will benefit the system. This paper presents an innovative model that provides an answer to this question and helps LDCs overcome the obstacles mentioned above.

This paper proposes a bus-wise incentive program (BWIP) that directs and encourages the integration of DG investments at targeted network locations where they will benefit the overall system. In other words, the total savings the LDC will realize through the implementation of DG projects will be managed wisely since a portion will be used for incentivizing DG owners and the rest will go into LDC coffers. Using the proposed model, the LDC also has the opportunity to identify the least cost solution from a combination of the proposed BWIP and traditional expansion options (i.e., upgrading or constructing new substations, upgrading or constructing new lines, and reconfiguring the system). It is this combination of choices that constitutes the innovative aspect of the novel IDSEP model, and in this way the model allows the LDC to coordinate its future expansion projects effectively with DG investors. The major player in this strategy is the LDC, while the DG investors are considered active followers. The proposed BWIP guarantees project feasibility and financial justification for the DG investors based on several economic indices. One of the major advantages of the proposed BWIP is that it can replace the Feed-In Tariff (FIT) program, which is currently being phased out in Ontario, and open the door for new DG polices whereby incentives are set up based on LDC and network needs.

As reported in the literature, the joint DSEP problem, in which DGs are incorporated as key alternatives in addition to conventional options, has been addressed through the introduction of a number of techniques and mathematical models [1], [3], [8]-[29]. Most of the research conducted in this regard has assigned the ownership of DGs to LDCs. For example, in the work described in [8], the distribution system was expanded by means of DG integration, system reconfiguration, switch installation, and rewiring. The possibility of performing dynamic planning based on a pseudo-dynamic procedure that included consideration of DGs as an alternative for LDCs was assessed in [9]. The authors of [3] and [10] explored several reinforcement techniques, such as dispatchable DGs, cross-connection feeders, and line and substation upgrades. Based on the assumed LDC ownership of the DGs, the objective was to minimize investment, operation, and reliability costs. The dynamic problem was solved using modified discrete particle swarm optimization: a significant reduction in transformer investment costs was observed. Similar work employing a genetic algorithm was reported in [11], with DGs, lines, and transformers considered as possible alternatives.

The same assumption underlies the study presented in [12], which involved the introduction of a heuristic method for distribution system expansion that utilizes dispatchable DGs, lines, and transformers. The required upgrade components and commissioning year were determined based on a benefit-to-cost ratio concept. Other researchers in [13] achieved two-level hierarchical distribution system planning that takes into account specific factors in a deregulated environment including regulatory policies, market prices, environmental considerations, and taxes. A joint expansion plan for distribution system networks and DG units was investigated in [14], [15]. Multistage long-term planning utilizing multiple alternatives such as voltage regulators, capacitor banks, and DGs was reported in [16]. In reference [17], the authors proposed a distribution system planning model in which all of the planning decisions in the primary and secondary distribution networks are coordinated. The use of low voltage feeders/substations, medium voltage feeders/substations, and medium voltage DGs represent planning alternatives for the green-field network. The authors in [18] expanded the distribution networks by means of DGs' integration and feeders' reinforcement. The multiyear planning aimed to minimize the investment, operation, and emission costs over the planning period. The deployment of renewable-based DGs was investigated in [19] as an option to reinforce the grid considering the reactive power capability for these DGs. A risk-based optimization method was proposed in [20] to implement DGs as flexible real options for the purpose of large network investments' deferment. A multiobjective distribution planning model was proposed in [21] to minimize the investment, operation, and emission costs incurred by LDCs. A heuristic-based technique was used to obtain the DG planning decisions and evaluate all system savings due to deferment of investments. Besides the lack of a proper inclusion of the relevant planning aspects (i.e. absence of uncertainty inclusion, static planning, heuristic-based solution, and deficiency of diverse planning options, as shown in Table I), all previous researches reviewed so far were in common based on the assumption that LDC is solely responsible for purchasing and operating the DGs which is impractical as it is stated earlier.

Some researchers have addressed the problem of DSEP by assuming that DG units belong to private investors. However, these models have been based on the assumptions that DG capacities, geographical locations, and capacity factors are known a priori (i.e. DGs are sized and allocated by investors initially), that the LDC has no control over such decisions which may lead to non-economical upgrade projects incurred by the LDC. Moreover, the bi-lateral financial agreements between DG investors as energy sellers and LDC as energy buyer are not considered, and that LDC and DG investor interaction is therefore nonexistent. For example, the authors in [22] determined the optimal sizes, quantities, and locations of distributed transformers and lines considering a three-phase power loss cost model in the objective function. However, the static model, which is solved heuristically, assumes DG locations and sizes are existing initially in the grid and there is no financial interaction between LDC and DG investors. The same assumptions and shortcomings underlie the research implemented in [23] which solves the distribution planning problem by combining modified load flow with graph theory based on a minimum spanning tree. The authors of [24] used an MILP model solved by simulated annealing in order to design a distribution system through a decomposition process. Another example in which LDC has no control over DG planning decisions, is the work presented in [25], which involved the coordination of multiple alternatives, including line/substation upgrades and capacitor bank/voltage regulator allocation. To carry out optimum multistage distribution system planning with DGs owned by investors, the authors of [26] extended the formal application of a linear disjunctive approach in their mathematical programming; however, the interaction between LDC and DG investors has not been considered. Based on the same previous assumptions and with a heuristic-based solution technique, the impact of microgrids (a group of renewable and non-renewable DGs as well as energy storages) on the planning of primary distribution networks is assessed in [27]. A generic planning framework to meet the requirements of LDCs in the United Kingdom for DGs integration is addressed in [28] with economic, engineering, and financial analyses.

A dynamic programming approach is utilized in [29] to expand the distribution system, and it is solved using genetic algorithm to obtain the network configuration. A multi-stage distribution system expansion planning-based reliability is employed in [1]. The problem is converted to a MILP problem utilizing piecewise linearization method to obtain the optimal planning configuration as well as feeder and substation capacities. However, DGs were not considered as planning options in these two studies. Table I presents a summary of the literature review considering all the planning aspects of the previous work and the planning features of the proposed model.

With the above discussion as background, this paper presents a novel long-term multistage IDSEP model of the DSEP problem that enables the LDC to establish bus-wise incentive prices for DG investors and to determine upgrade decisions for

TABLE I
SUMMARY OF LITERATURE REVIEW PLANNING FEATURES

Ref.	**DG ownership**		**Interaction with DG investors**	**Planning decisions and alternatives**						**Uncertainty**	**Planning period**	**Solution technique**
	LDC	Private Investor		DG size	DG location	DG incentive	Substation upgrade/ construct	Feeder upgrade/ construct	Modifying network topology			
[1]	NA	NA					√	√	√	NC	Dynamic	MILP-MP
[3], [9]-[13]	√		NC	√	√		√	√		NC	Dynamic	Heuristic
[8]	√		NC	√	√			√		Considered	Static	Heuristic
[14], [16]	√		NC	√	√		√	√	√	NC	Dynamic	MILP-MP
[15]	√		NC	√	√		√	√	√	Considered	Dynamic	MILP-MP
[17]	√		NC	√	√		√	√		NC	Static	Heuristic
[18]	√		NC	√	√			√		NC	Dynamic	Heuristic
[19]	√		NC	√	√					Considered	Static	Heuristic
[20]	√		NC	√	√					NC	Dynamic	Heuristic
[21]	√		NC	√	√			√	√	Considered	Dynamic	Heuristic
[22], [23]		√	NC				√	√	√	NC	Static	Heuristic
[24]		√	NC					√	√	NC	Static	Heuristic
[25]		√	NC				√	√	√	NC	Dynamic	Heuristic
[26]		√	NC				√	√	√	NC	Dynamic	MILP-MP
[27]	NA	NA						√		NC	Static	Heuristic
[29]	NA	NA						√		NC	Static	Heuristic
Proposed Work		√	Considered	√	√	√	√	√	√	Considered	Dynamic	MILP-MP

NA: Not applicable, NC: Not considered, MILP-MP: Mixed integer linear programing solved using mathematical based programming technique.

some of the distribution system assets. The new model invites and encourages DG investors to participate effectively and play a key role in reinforcement and expansion plans. The proposed active interaction between the LDC and DG investors is represented through long or mid-term contracts in which the DG investors are committed to install and operate their DG projects at specific locations and capacities determined by the LDC, whereas the LDC is committed to buy all of the energy generated by these projects at guaranteed prices (incentives) for the full periods of the contracts. Therefore, both parties benefit from this practice with the LDC experiencing substantial savings due to reduced operating and running costs as well as the elimination or deferment of massive infrastructure upgrade plans, and the DG investors investing in such projects wherein their profitability and returns are guaranteed. The proposed model also allows the LDC to identify the least cost solution obtainable from a combination of traditional upgrade alternatives and the proposed BWIP undertaken with the DGs. An additional feature is comprehensive uncertainty modeling that addresses the stochastic nature of system demand and of the output power produced by renewable-based DGs.

The primary contributions of the work presented in this paper are fourfold:

1) The proposed incentive-based DSEP (IDSEP) model will help an LDC define necessary expenditures while also implementing a BWIP to encourage the integration of DG projects at specific buses that will benefit the system. The following are the key features of the proposed IDSEP model:

a) It determines the time, location, capacity, technology, and incentive price for each DG investment.

b) It determines the commissioning year and capacity for the required distribution component upgrade plans to be undertaken by the LDC. This may include upgrading existing substations, constructing new substations, upgrading existing lines, building new lines, or modifying the network topology.

c) The bus-wise incentive program is more efficient than most regulations whose provisions apply identical incentive prices for all buses.

d) As a FIT program is phased out, as in Ontario, this model can function as a replacement that allows LDCs to determine incentive prices and appropriate DG locations based on their requirements and system needs.

2) A comprehensive methodology is presented for modeling the intermittent behavior of both fluctuating demand and the power generated from wind and PV-based DGs.

3) Profitability for DG investors is ensured through the assessment and consideration of a variety of economic indices. The model incorporates the most popular financial-based indicators for DG investors including internal rate of return, profit investment ratio, and discounted payback period.

4) Several linearization techniques are presented to transform the proposed IDSEP model from MINLP into MILP model in which the convergence to optimality is guaranteed. These linearization methods can be applied to any planning and operation problems.

The remainder of the paper is organized as follows: Section II describes the modeling of the uncertainty associated with the load and with DG components. The proposed problem formulation for the IDSEP model is introduced in Section III. Section IV presents the linearization methods used in the paper. Section V reports the numerical results for the case studies conducted, and Section VI summarizes the study, presents conclusions, and reiterates the primary contributions.

II. MODELING OF THE UNCERTAINTY ASSOCIATED WITH DEMAND AND DG OUTPUT POWER

Constructing a suitable model that can capture the intermittent behavior resulting from the stochastic nature of wind- and PV-based DG output power and of fluctuations in the demand has become imperative. This factor was a primary consideration in the development of the proposed probabilistic IDSEP model. The study presented in this paper involved the generation of a multi-scenario-based model in which renewable DG output power and power demand are treated probabilistically. The uncertainty modeling entailed the following steps:

1) Five successive years of historical wind speed, solar irradiance, and system demand data are collected.

2) For each data type, several probability distribution functions are examined in order to determine the best distribution that fits each data type. Based on the methods commonly reported in the literature for modeling the uncertainty of wind speed, solar irradiance, and power demand, five distribution functions are tested: Weibull, Normal, Rayleigh, Gamma, and Lognormal [30]. Kolmogorov-Smirnov algorithm (K-S) is applied to find the best fit for each data type [30], [31]. The methodology of this method consists of the following steps:

a) The parameters of the probability density functions are defined using the mean v_m and standard deviation v_σ of the data. For example, the shape index k and scale index sc of the Weibull distribution can be obtained using (1) and (2), as in [30], [32]:

$$k = \left(\frac{v_\sigma}{v_m}\right)^{-1.086} \tag{1}$$

$$sc = \frac{v_m}{\Gamma\left(1+\frac{1}{k}\right)} \tag{2}$$

b) The cumulative distribution function (CDF) for each distribution is constructed using the parameters obtained in step a. For example, the Weibull distribution CDF is given in (3):

$$CDF(v_m) = 1 - e^{-\left(\frac{v_m}{sc}\right)^k} \tag{3}$$

The empirical cumulative distribution function (ECDF) of the data is then constructed.

c) The mean absolute error (MAE) is next computed for each probability distribution. The value of each MAE is equal to the summation of the differences between the data points on the ECDF and on the CDF over the total number of data points TP, as defined in (4):

$$MAE = \frac{\sum_{v_m=0}^{TP} |CDF(v_m) - ECDF(v_m)|}{TP} \tag{4}$$

The distribution function that has the minimum MAE for each data type is ultimately chosen as representing that type. Three distribution functions are thus selected for modeling wind and PV output power plus system demand.

3) Once the probability distribution functions for wind speed, solar radiation, and system demand are defined, these PDFs must be divided into many states for incorporation into the calculations. Depending on the maximum value and how many intervals are required, the PDFs are divided into multiple equal intervals. The size of each state is dependent on the number of intervals required N_b, the mean m, and the standard deviation S. The value of each state is represented by the midpoint of each interval $MB_{int}(r)$, as indicated in equation (5) where r is an index for the intervals [30]:

$$MB_{int}(r) = \begin{cases} m + \left(\frac{10S}{N_b}\right)(r - 0.5N_b); & odd\ N_b \\ m + \left(\frac{10S}{N_b}\right)(r - 0.5(N_b+1)); & even\ N_b \end{cases} \tag{5}$$

The probability for each state can be obtained using the integral equation (6):

$$P(y_a \le y \le y_b) = \int_{y_a}^{y_b} f(y).\,dy \tag{6}$$

where y_a and y_b are the starting and ending variables for state y, respectively, and $f(y)$ is the probability density function of the selected distribution.

4) The per unit values of the output power produced from wind- and PV-based DGs are then computed using the applicable equations from (7)-(12). In the case of wind power, per unit output power for each state is calculated using the following equation [30], [32]:

$$OP_w(v_{ay}) = \begin{cases} 0 & 0 \le v_{ay} \le v_{ci}, v_{ay} \ge v_{co} \\ P_{rated} \times \frac{v_{ay} - v_{ci}}{v_r - v_{ci}} & v_{ci} \le v_{ay} \le v_r \\ P_{rated} & v_r \le v_{ay} \le v_{co} \end{cases} \tag{7}$$

where v_{ci}, v_r, v_{co} are the cut-in speed, rated speed, and cut-off speed of the wind turbine, respectively; $OP_w(v_{ay})$ is the output power during state *y*; and v_{ay} is the average speed of state *y*.

The PV per unit output power for each state is calculated using the following equations [30], [32]:

$$T_{C_y} = T_A + s_{ay}\left(\frac{N_{OT} - 20}{0.8}\right) \tag{8}$$

$$I_y = s_{ay}[I_{sc} + K_i(T_c - 25)] \tag{9}$$

$$V_y = V_{oc} - K_v T_{C_y} \tag{10}$$

$$FF = \frac{V_{MMP} \times I_{MMP}}{V_{oc} \times I_{sc}} \tag{11}$$

$$OP_{PV}(s_{ay}) = N_m \times FF \times I_y \times V_y \tag{12}$$

where T_{C_y} is the cell temperature, in °C, during state *y*; T_A is the ambient temperature, in °C; K_v is the voltage temperature coefficient *V/C*; K_i is the current temperature coefficient *A/C*; N_{OT} is the nominal operating temperature of the cell, in °C; FF is the fill factor; N_m is the number of modules; I_{sc} is the short circuit current, in A; V_{oc} is the open circuit voltage, in V; I_{MMP} is the current at maximum power point, in A; V_{MMP} is the voltage at maximum power point, in V; $OP_{PV}(s_{ay})$ is the per unit output power during state *y*; and s_{ay} is the average irradiance of state *y*.

5) After all states for wind power, solar power, and system load are defined, a three-column matrix that includes all possible combinations (scenarios) of the states is created, in which column 1 represents the wind-based DG output power states (p.u.), column 2 represents the solar DG output power states (p.u.), and column 3 represents the different load states or levels (p.u.). This multi-scenario matrix has rows equal to the total number of overall scenarios, which is equal to the multiplication of wind, solar, and load states. The probability of each scenario is equal to the product of the wind state probability, solar state probability, and load state probability for that corresponding scenario,

wherein wind speed, solar irradiance, and load are assumed to be independent events.

Regarding the variation of energy prices over the planning horizon, the energy prices are forecasted using Autoregressive Moving Average (ARIMA) model [33]. As the energy prices are closely tied to demand change [34], the average energy price for each demand state at each year in the planning period is forecasted. Then, these prices are matched according to the corresponding demand state (i.e. the states that have low demand will share a similar average energy price, and likewise for other states).

III. IDSEP Model Problem Formulation

This section presents the proposed multistage IDSEP model, which includes consideration of the payments made by the LDC to encourage DG connection at the specific buses that will ensure the financial justification of the DG projects. Also considered are all investment and operation costs for new and existing alternatives. The overall objective is thus to identify the minimum overall planning costs by taking into account all of the above components; establishing the BWIP prices for different types of DGs; and determining the optimal sites, sizes, times, and technologies for any additions, both new generation and upgrades to existing assets. The scope of the work presented in this paper is concerning the primary distribution systems with high/medium substations and medium voltage feeders. Fig. 1 illustrates the flowchart of the proposed IDSEP model.

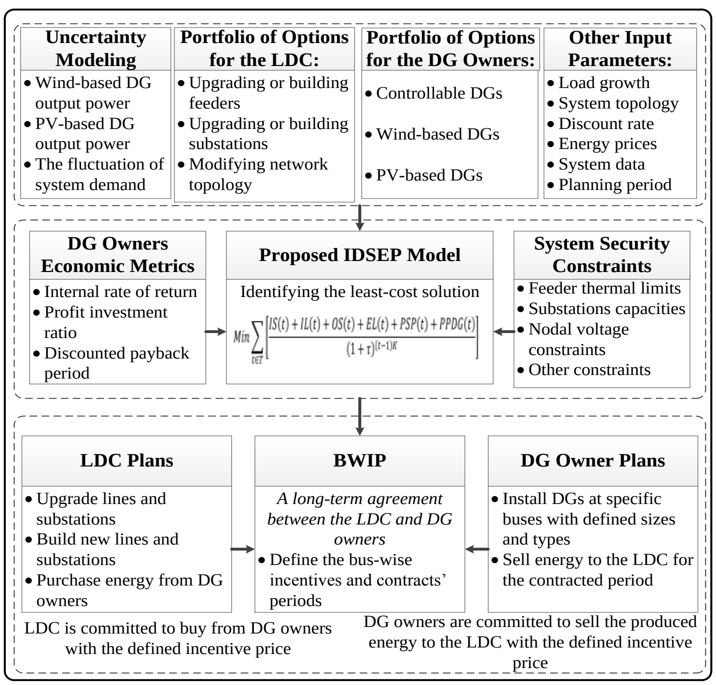

Fig. 1. Flowchart of the proposed IDSEP model.

A. Objective Function

The objective function is comprised of all investment and operation costs incurred by the LDC. The components of the objective function are the substation investment (IS), the line investment (IL), the substation operation cost (OS), the cost of energy loss (EL), the energy purchased from the market (PSP), and the energy purchased from the DG investors (PPDG). The mathematical formulation of the objective function is as follows:

$$Min \sum_{t\in T}\left[\frac{IS(t)+IL(t)+OS(t)+EL(t)+PSP(t)+PPDG(t)}{(1+\tau)^{(t-1)K}}\right] \tag{13}$$

The mathematical formulations for the components of the objective function are shown in (14)-(19).

$$IS(t)=\sum_{i\in\Omega_{ES}}\sum_{u\in\Omega_U}\left(C_u^{US}\sigma_{i,u,t}\right)+\sum_{j\in\Omega_{CS}}\sum_{c\in\Omega_C}\left(C_c^{NS}u_{j,c,t}\right) \tag{14}$$

$$IL(t)=\sum_{ij\in\Omega_{EL}}\sum_{a\in\Omega_a}C_a^{UF}L_{ij}\beta_{ij,a,t}+\sum_{ij\in\Omega_{CL}}\sum_{a\in\Omega_a}C_a^{NF}L_{ij}z_{ij,a,t} \tag{15}$$

$$OS(t)=\sum_{i\in\Omega_{ES}}\sum_{e\in\Omega_{se}}\left(S_{G_{i,e,t}}^{sqr}\alpha_e\varphi\omega\right)f(\tau,K)+\sum_{j\in\Omega_{CS}}\sum_{e\in\Omega_{se}}\left(S_{G_{j,e,t}}^{sqr}\alpha_e\varphi\omega\right)f(\tau,K) \tag{16}$$

$$EL(t)=\sum_{ij\in\Omega_{EL}}\sum_{e\in\Omega_{se}}\left(Pl_{ij,e,t}\alpha_e\varphi\varepsilon\right)f(\tau,K)+\sum_{ij\in\Omega_{CL}}\sum_{e\in\Omega_{se}}\left(Pl_{ij,e,t}\alpha_e\varphi\varepsilon\right)f(\tau,K) \tag{17}$$

$$PSP(t)=\sum_{i\in\Omega_{ES}}\sum_{e\in\Omega_{se}}\left(P_{G\,i,e,t}\alpha_e\varphi C_{e,t}^{E}\right)f(\tau,K)+\sum_{j\in\Omega_{CS}}\sum_{e\in\Omega_{se}}\left(P_{G\,j,e,t}\alpha_e\varphi C_{e,t}^{E}\right)f(\tau,K) \tag{18}$$

$$PPDG(t)=\sum_{dg\in\Omega_{DG}}\sum_{i\in\Omega_N}\sum_{e\in\Omega_{se}}\rho_{dg}\left(INC_{dg,i,t}C_{dg,e}\alpha_e\varphi\right)f(\tau,K) \tag{19}$$

The function $f(\tau,K)=\left(\frac{1-(1+\tau)^{-K}}{\tau}\right)$ is called the present value of annuity function, which calculates the present value of a series of future constant annualized payments at a given time.

B. Power Conservation Constraints

In each node in the distribution system, active and reactive power flow must be balanced as in (20) and (21). The parameter $\epsilon_{dg}=\left(\frac{\sin(\arccos(pf_{dg}))}{pf_{dg}}\right)$ in (21) is used for calculating the DG reactive power as a function of the DG active power using the DG power factor (pf_{dg}). Equations (22) and (23) represent the active and reactive power flows associated with line ij as a function of nodal voltages and nodal voltage angles. They are represented as nonlinear functions multiplied by the feeder utilization binary variable so that, if the feeder is on service or needs to be built, the binary variable equals one. Otherwise, this binary value will be zero.

$$P_{G\,i,e,t}+\sum_{dg\in\Omega_{DG}}\rho_{dg}OP_{dg,e}P_{DG\,dg,i,t}-DL_eP_{D\,i,t}-\sum_{ij\in\Omega_L}P_{ij,e,t}+\sum_{ki\in\Omega_L}P_{ki,e,t}-\sum_{ij\in\Omega_L}Pl_{ij,e,t}=0\quad\forall i\in\Omega_N,e\in\Omega_{se},t\in T \tag{20}$$

$$Q_{G\,i,e,t}+\sum_{dg\in\Omega_{DG}}\rho_{dg}OP_{dg,e}\epsilon_{dg}P_{DG\,dg,i,t}-DL_eQ_{D\,i,t}-\sum_{ij\in\Omega_L}Q_{ij,e,t}+\sum_{ki\in\Omega_L}Q_{ki,e,t}-\sum_{ij\in\Omega_L}Ql_{ij,e,t}=0\quad\forall i\in\Omega_N,e\in\Omega_{se},t\in T \tag{21}$$

$$P_{ij,e,t}=x_{ij,t}\left(V_{i,e,t}^2G_{ij}-V_{i,e,t}V_{j,e,t}G_{ij}\cos\left(\delta_{i,e,t}-\delta_{j,e,t}\right)-V_{i,e,t}V_{j,e,t}B_{ij}\sin\left(\delta_{i,e,t}-\delta_{j,e,t}\right)\right)\quad\forall ij\in\Omega_L,e\in\Omega_{se},t\in T \tag{22}$$

$$Q_{ij,e,t} = x_{ij,t}\big(-V_{i,e,t}^2 B_{ij} - V_{i,e,t}V_{j,e,t}G_{ij}\sin(\delta_{i,e,t}-\delta_{j,e,t}) + V_{i,e,t}V_{j,e,t}B_{ij}\cos(\delta_{i,e,t}-\delta_{j,e,t})\big) \quad \forall ij \in \Omega_L, e \in \Omega_{se}, t \in T \tag{23}$$

C. Other Constraints

This section itemizes other planning constraints.

1) Active and Reactive Power Losses:

$$Pl_{ij,e,t} = x_{ij,t}G_{ij}\big(V_{i,e,t}^2 + V_{j,e,t}^2 - 2V_{i,e,t}V_{j,e,t}\cos(\delta_{j,e,t}-\delta_{i,e,t})\big) \quad \forall ij \in \Omega_L, e \in \Omega_{se}, t \in T \tag{24}$$

$$Ql_{ij,e,t} = -x_{ij,t}B_{ij}\big(V_{i,e,t}^2 + V_{j,e,t}^2 - 2V_{i,e,t}V_{j,e,t}\cos(\delta_{j,e,t}-\delta_{i,e,t})\big) \quad \forall ij \in \Omega_L, e \in \Omega_{se}, t \in T \tag{25}$$

2) Substation Capacity Constraints: Equation (26) ensures that the square of the apparent power drawn from the existing substation must be lower than or equal to the existing substation capacity plus the substation upgrade decision. If there is no need to upgrade the substation, the second term on the right side of (26) must be zero. Equation (27) represents the limit on the power drawn from the candidate substation and basically defines the required capacity of the new candidate substation. The square of the apparent power drawn from the substation as a function in the substation's active and reactive power is shown in (28).

$$S_{G_{i,e,t}}^{sqr} \le \left(\overline{S_{G_i}^{max}}\right)^2 + \sum_{u\in\Omega_U}\sum_{t'=1}^{t}(S_u^{US})^2\sigma_{i,u,t} \quad \forall i \in \Omega_{ES}, e \in \Omega_{se}, t \in T \tag{26}$$

$$P_{G_{j,e,t}}^2 + Q_{G_{j,e,t}}^2 \le \sum_{c\in\Omega_C}\sum_{t'=1}^{t}(S_c^{NS})^2 u_{j,c,t} \quad \forall j \in \Omega_{CS}, e \in \Omega_{se}, t \in T \tag{27}$$

$$S_{G_{i,e,t}}^{sqr} = P_{G_{i,e,t}}^2 + Q_{G_{i,e,t}}^2 \quad \forall i \in \Omega_{SS}, e \in \Omega_{se}, t \in T \tag{28}$$

3) Feeder Flow and Thermal Capacity Limits: Equation (29) ensures that the current flow in the feeder is within the thermal capacity of the feeder. If upgrading this feeder is essential, the second term on the right side of (29) covers that contingency by replacing the old feeder with the new one. Equation (30) is responsible for decisions related to the construction of any new candidate feeders. The square of the apparent power flowing in feeder *ij* as a function in the feeder's active and reactive power is shown in (31).

$$S_{ij,e,t}^{sqr} \le \left(\overline{S_{ij}^{max}}\right)^2\left(1 - \sum_{a\in\Omega_a}\sum_{t'=1}^{t}\beta_{ij,a,t}\right) + \sum_{a\in\Omega_a}\sum_{t'=1}^{t}(S_a^P)^2\beta_{ij,a,t} \quad \forall ij \in \Omega_{EL}, e \in \Omega_{se}, t \in T \tag{29}$$

$$S_{ij,e,t}^{sqr} \le \sum_{a\in\Omega_a}\sum_{t'=1}^{t}(S_a^P)^2 z_{ij,a,t} \quad \forall ij \in \Omega_{CL}, e \in \Omega_{se}, t \in T \tag{30}$$

$$S_{ij,e,t}^{sqr} = P_{ij,e,t}^2 + Q_{ij,e,t}^2 \quad \forall ij \in \Omega_L, e \in \Omega_{se}, t \in T \tag{31}$$

4) Bus Voltage Constraint: The voltage magnitude in each system bus must be kept within permissible voltage limits, as set out in (32):

$$\underline{V}^{Min} \le V_{i,e,t} \le \overline{V}^{Max} \quad \forall i \in \Omega_N, e \in \Omega_{se}, t \in T \tag{32}$$

5) LDC Investment Decision Constraints: Equations (33)-(36) ensure that any upgrade decision for a feeder/substation and any construction decision for a feeder/substation must be executed once over the planning horizon.

$$\sum_{u\in\Omega_U}\sum_{t\in T}\sigma_{i,u,t} \le 1 \quad \forall i \in \Omega_{ES} \tag{33}$$

$$\sum_{c\in\Omega_C}\sum_{t\in T}u_{j,c,t} \le 1 \quad \forall j \in \Omega_{CS} \tag{34}$$

$$\sum_{a\in\Omega_a}\sum_{t\in T}\beta_{ij,a,t} \le 1 \quad \forall ij \in \Omega_{EL} \tag{35}$$

$$\sum_{a\in\Omega_a}\sum_{t\in T}z_{ij,a,t} \le 1 \quad \forall ij \in \Omega_{CL} \tag{36}$$

6) System Radiality Constraint: Most existing distribution systems have a radial configuration due to the simplicity of operation and the coordination of radial topology protection. Maintaining this topology during planning and operation processes is therefore crucial. Equation (37) is used for preventing any loop in the network and for maintaining the radial topology, based on the definition of the graph tree as in [35].

$$\sum_{ij\in\Omega_L}x_{ij,t} = N_b - N_{ES} - \sum_{j\in\Omega_{CS}}\sum_{c\in\Omega_C}u_{j,c,t} \quad \forall t \in T \tag{37}$$

7) DG Investment and Utilization Constraints: To direct DG investors to integrate their DGs at specific locations, the LDC should provide bus-wise incentives that guarantee profitability for the DG investors. Due to the high investment costs for such DG projects and different economic perspectives for the investors, it is necessary to analyze and address a verity of economic indicators for that kind of investments. For example, if the DG owners are more interested in the amount of value created per unit of investment, they may use the profit investment ratio to quantify that. Some investors are concerned about the money liquidity and when the project pays off its costs to utilize that money for starting other projects. In this case, discounted payback period is the best way to assist DG owners for that matter. Furthermore, if the investors are interested in the percentage rate earned on each dollar spent along the project period, they may use internal rate of return-based indicator. Therefore, a number of economic indices, namely IRR, PIR, and DPP, are considered in order to ensure the feasibility of an investment with respect to investment and operation costs as well as overall benefit for the DG.

For each bus in the system, equations (38) and (39) determine the total DG investment and operation costs, and equation (40) calculates the total benefit accruing to the DG investors when they sell the energy produced at the incentive price. As explained earlier, the function $f(IRR_{dg,i},K) = \left(\frac{1-(1+IRR_{dg,i})^{-K}}{IRR_{dg,i}}\right)$ in (39) and (40) is used for determining the present annuity value. The incentive cost is formulated in (41)

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as a multiplication of DG power and bus-wise incentive price (BWIP).

$$INVC_{dg,i,t}^{DG} = \rho_{dg}\left(P_{DG\,dg,i,t} - P_{DG\,dg,i,t-1}\right) C_{dg}^{IDG} \quad \forall dg \in \Omega_{DG}, i \in \Omega_N, t \in T \tag{38}$$

$$OPC_{dg,i,e,t}^{DG} = \rho_{dg}\left(C_{dg,e} P_{DG\,dg,i,t} C_{dg}^{ODG} \alpha_e \varphi\right) f\left(IRR_{dg,i}, K\right) \quad \forall dg \in \Omega_{DG}, i \in \Omega_N, e \in \Omega_{se}, t \in T \tag{39}$$

$$BEN_{dg,i,e,t}^{DG} = \rho_{dg}\left(C_{dg,e} INC_{dg,i,t} \alpha_e \varphi\right) f\left(IRR_{dg,i}, K\right) \quad \forall dg \in \Omega_{DG}, i \in \Omega_N, e \in \Omega_{se}, t \in T \tag{40}$$

$$INC_{dg,i,t} = P_{DG\,dg,i,t} \gamma_{dg,i} \quad \forall dg \in \Omega_{DG}, i \in \Omega_N, t \in T \tag{41}$$

Equations (42)-(44) compute the present values of DG installation and operation costs as well as the DG benefit at each bus in the network. These values will be used to calculate the economic metrics of the DG projects.

$$PVINV_{dg,i}^{DG} = \sum_{t \in T} INVC_{dg,i,t}^{DG} \left(1 + IRR_{dg,i}\right)^{-(t-1)K} \quad \forall dg \in \Omega_{DG}, i \in \Omega_N \tag{42}$$

$$PVOPE_{dg,i}^{DG} = \sum_{t \in T} \sum_{e \in \Omega_{se}} OPC_{dg,i,e,t}^{DG} \left(1 + IRR_{dg,i}\right)^{-(t-1)K} \quad \forall dg \in \Omega_{DG}, i \in \Omega_N \tag{43}$$

$$PVBEN_{dg,i}^{DG} = \sum_{t \in T} \sum_{e \in \Omega_{se}} BEN_{dg,i,e,t}^{DG} \left(1 + IRR_{dg,i}\right)^{-(t-1)K} \quad \forall dg \in \Omega_{DG}, i \in \Omega_N \tag{44}$$

a) Internal rate of return and minimum acceptable rate of return: Widely used for assessing the attractiveness of a project, the internal rate of return (IRR) is a metric that basically represents the interest rate at which the net present value (NPV) of all cash flows from a project becomes zero. This metric is usually compared with the hurdle rate, or minimum acceptable rate of return (MARR) initially specified by the investor. If the IRR is greater than or equal to the MARR, then the project is considered profitable, and the investor would therefore accept the project. Equation (45) ensures that the NPV of all cash flows equals zero, taking into consideration that the IRR of each project is equal to the MARR of that corresponding project.

$$PVINV_{dg,i}^{DG} + PVOPE_{dg,i}^{DG} - PVBEN_{dg,i}^{DG} = 0 \quad \forall dg \in \Omega_{DG}, i \in \Omega_N \tag{45}$$

b) Profit investment ratio: The second economic metric used in this work is the profit investment ratio (PIR), or the profitability index (PI). This index measures the ratio between the present value of the gain or benefit to be derived from an investment and the present value of the cost of the investment. If the PI is greater than one, the NPV of the project is positive, and the project will thus be accepted. A DG investor may also state an acceptable PI, which should be constrained in the planning, as expressed in (46).

$$PVBEN_{dg,i}^{DG} \geq PIR\left(PVINV_{dg,i}^{DG} + PVOPE_{dg,i}^{DG}\right) \quad \forall dg \in \Omega_{DG}, i \in \Omega_N \tag{46}$$

c) Discounted payback period: The payback period defines the length of time (typically in years) at the end of which the project will recoup or recover the cost of the investment. The discounted payback period (DPP) incorporates a discount rate for taking into account the time value of money. The DPP metric is not normally used for evaluating project feasibility since it ignores all incoming cash flows that follow the breakeven point. In the work presented in this paper, DPP is calculated after the planning outcomes are obtained so that it is not included in the optimization. Equation (47) calculates the DPP of the DG projects at each bus:

$$DPP_{dg,i} = Y_{NN} + \frac{\left|CCF_{Y_{NN}}\right|}{CCF_{Y_{NN}+1} + \left|CCF_{Y_{NN}}\right|} \quad \forall dg \in \Omega_{DG}, i \in \Omega_N \tag{47}$$

where Y_{NN} is the year in which the last negative value of the cumulative discounted cash flow occurs, $CCF_{Y_{NN}}$ is the last negative value of the cumulative discounted cash flow, and $CCF_{Y_{NN}+1}$ is the first positive value of the cumulative discounted cash flow.

8) DG Penetration Constraints: The maximum DG capacity that can be connected to any bus in the network is constrained as in (48), a limit based on technical studies conducted by the LDC. Equation (49) ensures that the penetration level of each renewable-based DG in the last stage of planning conforms with environmental regulation requirements.

$$\sum_{dg \in \Omega_{DG}} \rho_{dg} P_{DG\,dg,i,t} \leq \overline{DG_i} \quad \forall i \in \Omega_N, t \in T \tag{48}$$

$$\sum_{i \in \Omega_N} \rho_{dg} P_{DG\,dg,i,t} \geq \mu \sum_{i \in \Omega_N} P_{D\,i,t} \quad \forall dg \in \Omega_{DG} \backslash \{CDG\}, \forall t = LT \tag{49}$$

9) DG Dynamic Constraint: The dynamic constraint denoted in (50) governs cumulative DG capacities between planning stages:

$$P_{DG\,dg,i,t} - P_{DG\,dg,i,t+1} \leq 0 \quad \forall dg \in \Omega_{DG}, i \in \Omega_N, t \in T \tag{50}$$

10) Incentive Prices Constraint: Incentive prices should be constrained with respect to minimum and maximum values (51):

$$\underline{\gamma} \leq \gamma_{dg,i} \leq \overline{\gamma} \quad \forall dg \in \Omega_{DG}, i \in \Omega_N \tag{51}$$

11) Binary Variables Constraints:

$$\sigma_{i,u,t} \in \{0,1\} \quad \forall i \in \Omega_{ES}, u \in \Omega_U, t \in T \tag{52}$$

$$u_{j,c,t} \in \{0,1\} \quad \forall j \in \Omega_{CS}, c \in \Omega_C, t \in T \tag{53}$$

$$\beta_{ij,a,t} \in \{0,1\} \quad \forall ij \in \Omega_{EL}, a \in \Omega_a, t \in T \tag{54}$$

$$z_{ij,a,t} \in \{0,1\} \quad \forall ij \in \Omega_{CL}, a \in \Omega_a, t \in T \tag{55}$$

$$x_{ij,t} \in \{0,1\} \quad \forall ij \in \Omega_L, t \in T \tag{56}$$

IV. Linearization of the IDSEP Model

The mathematical model of the proposed IDSEP is described by (13)-(56). However, this model is MINLP due to the non-linearity of some constraints and expressions (i.e. equations (22)-(25), (28), (31), and (41)). In order to obtain a robust and efficient model, the non-linear expressions are linearized in this section; thus, the IDSEP model is converted from MINLP to MILP.

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A. Linearization of Equations (22) and (23)

The power flow equations explained in (22) and (23) are approximated by considering two valid practical assumptions. The first assumption is that the voltage magnitude at each bus is very close to 1 p.u.; thus, the bus voltages can be rewritten as a sum of 1 p.u. and small voltage deviation ($V_{i,e,t} = 1 + \Delta V_{i,e,t}$). The second assumption is that the angle difference across a line is very small so that the approximations $\cos(\delta_{i,e,t} - \delta_{j,e,t}) \approx 1$ and $\sin(\delta_{i,e,t} - \delta_{j,e,t}) \approx \delta_{i,e,t} - \delta_{j,e,t}$ can be applied. Therefore, equations (22) and (23) can be approximated as follows:

$$P_{ij,e,t} \cong x_{ij,t}\left((\Delta V_{i,e,t} - \Delta V_{j,e,t})G_{ij} - (\delta_{i,e,t} - \delta_{j,e,t})B_{ij}\right) \quad \forall ij \in \Omega_L, e \in \Omega_{se}, t \in T \tag{57}$$

$$Q_{ij,e,t} \cong x_{ij,t}\left(-(\Delta V_{i,e,t} - \Delta V_{j,e,t})B_{ij} - (\delta_{i,e,t} - \delta_{j,e,t})G_{ij}\right) \quad \forall ij \in \Omega_L, e \in \Omega_{se}, t \in T \tag{58}$$

$$\underline{\Delta V_i}^{Min} \le \Delta V_{i,e,t} \le \overline{\Delta V_i}^{Max} \qquad \forall i \in \Omega_N, e \in \Omega_{se}, t \in T \tag{59}$$

The full approximation steps can be found in [36]. However, equations (57) and (58) are still non-linear due to the bilinear product of the feeder utilization binary and voltage and angle variables. This non-linearity can be avoided by using the big-M formulation as follows:

$$(x_{ij,t} - 1)M \le P_{ij,e,t} - \left((\Delta V_{i,e,t} - \Delta V_{j,e,t})G_{ij} - (\delta_{i,e,t} - \delta_{j,e,t})B_{ij}\right) \le (1 - x_{ij,t})M \quad \forall ij \in \Omega_L, e \in \Omega_{se}, t \in T \tag{60}$$

$$(x_{ij,t} - 1)M \le Q_{ij,e,t} - \left(-(\Delta V_{i,e,t} - \Delta V_{j,e,t})B_{ij} - (\delta_{i,e,t} - \delta_{j,e,t})G_{ij}\right) \le (1 - x_{ij,t})M \quad \forall ij \in \Omega_L, e \in \Omega_{se}, t \in T \tag{61}$$

$$-\overline{S_{ij}^{max}}x_{ij,t} \le P_{ij,e,t} \le \overline{S_{ij}^{max}}x_{ij,t} \qquad \forall ij \in \Omega_L, e \in \Omega_{se}, t \in T \tag{62}$$

$$-\overline{S_{ij}^{max}}x_{ij,t} \le Q_{ij,e,t} \le \overline{S_{ij}^{max}}x_{ij,t} \qquad \forall ij \in \Omega_L, e \in \Omega_{se}, t \in T \tag{63}$$

B. Linearization of Equations (24) and (25)

By following the same two assumptions above and neglecting the higher order terms, the active and reactive power losses can be rewritten as follows:

$$Pl_{ij,e,t} = x_{ij,t}R_{ij}S_{ij,e,t}^{sqr} \qquad \forall ij \in \Omega_L, e \in \Omega_{se}, t \in T \tag{64}$$

$$Ql_{ij,e,t} = x_{ij,t}X_{ij}S_{ij,e,t}^{sqr} \qquad \forall ij \in \Omega_L, e \in \Omega_{se}, t \in T \tag{65}$$

Researchers are referred to reference [37] for the full derivation of equations (64) and (65). Equations (64) and (65) are still non-linear due to the presence of bilinear product. This issue is avoided by using the big-M method as follows:

$$(x_{ij,t} - 1)M \le Pl_{ij,e,t} - R_{ij}S_{ij,e,t}^{sqr} \le (1 - x_{ij,t})M \qquad \forall ij \in \Omega_L, e \in \Omega_{se}, t \in T \tag{66}$$

$$(x_{ij,t} - 1)M \le Ql_{ij,e,t} - X_{ij}S_{ij,e,t}^{sqr} \le (1 - x_{ij,t})M \qquad \forall ij \in \Omega_L, e \in \Omega_{se}, t \in T \tag{67}$$

$$-\overline{S_{ij}^{max}}x_{ij,t} \le Pl_{ij,e,t} \le \overline{S_{ij}^{max}}x_{ij,t} \qquad \forall ij \in \Omega_L, e \in \Omega_{se}, t \in T \tag{68}$$

$$-\overline{S_{ij}^{max}}x_{ij,t} \le Ql_{ij,e,t} \le \overline{S_{ij}^{max}}x_{ij,t} \qquad \forall ij \in \Omega_L, e \in \Omega_{se}, t \in T \tag{69}$$

C. Linearization of Equation (28)

The quadratic expressions of the right member of equation (28) can be linearized by using piecewise linearization with sufficient linear segments or blocks Y as in [16]. Therefore, equation (28) can be rewritten as:

$$S_{G_{i,e,t}}^{sqr} \cong \sum_{y=1}^{Y}(2y-1)\bar{\Delta}^G \Delta P_{G_{i,e,t,y}} + \sum_{y=1}^{Y}(2y-1)\bar{\Delta}^G \Delta Q_{G_{i,e,t,y}} \quad \forall i \in \Omega_{SS}, e \in \Omega_{se}, t \in T \tag{70}$$

The active and reactive powers drawn from the substations are expressed as a sum of a series of linear segments $\Delta P_{G_{i,e,t,y}}$ and $\Delta Q_{G_{i,e,t,y}}$, respectively, as shown in (71) and (72). The discretization variables for the active and reactive power are constrained, as in (73) and (74), while equation (75) defines the value used for discretization.

$$P_{G_{i,e,t}} = \sum_{y=1}^{Y} \Delta P_{G_{i,e,t,y}} \qquad \forall i \in \Omega_{SS}, e \in \Omega_{se}, t \in T \tag{71}$$

$$Q_{G_{i,e,t}} = \sum_{y=1}^{Y} \Delta Q_{G_{i,e,t,y}} \qquad \forall i \in \Omega_{SS}, e \in \Omega_{se}, t \in T \tag{72}$$

$$\Delta P_{G_{i,e,t,y}} \le \bar{\Delta}^G \quad \forall i \in \Omega_{SS}, e \in \Omega_{se}, t \in T, y \in Y \tag{73}$$

$$\Delta Q_{G_{i,e,t,y}} \le \bar{\Delta}^G \quad \forall i \in \Omega_{SS}, e \in \Omega_{se}, t \in T, y \in Y \tag{74}$$

$$\bar{\Delta}^G = \frac{\overline{V}^{Max}}{Y} max\{S_u^{US}, u \in \Omega_U\} \tag{75}$$

D. Linearization of Equation (31)

The linearization process in this section is similar to the method applied previously in section IV-C. By using the piecewise linearization, equation (31) can be approximated as follows:

$$S_{ij,e,t}^{sqr} \cong \sum_{y=1}^{Y}(2y-1)\bar{\Delta}^L \Delta P_{ij,e,t,y} + \sum_{y=1}^{Y}(2y-1)\bar{\Delta}^L \Delta Q_{ij,e,t,y} \quad \forall ij \in \Omega_L, e \in \Omega_{se}, t \in T \tag{76}$$

The active and reactive power flows in the feeder are expressed using non-negative auxiliary variables to obtain their absolute values as in (77) and (78). Also, the active and reactive power flows in feeder ij are expressed as a sum of a series of linear segments $\Delta P_{ij,e,t,y}$ and $\Delta Q_{ij,e,t,y}$, respectively, as shown in (79) and (80). The discretization variables are constrained as in (81) and (82), while equation (83) defines the value used for discretization.

$$P_{ij,e,t} = P_{ij,e,t}^{+} - P_{ij,e,t}^{-} \qquad \forall ij \in \Omega_L, e \in \Omega_{se}, t \in T \tag{77}$$

$$Q_{ij,e,t} = Q_{ij,e,t}^{+} - Q_{ij,e,t}^{-} \qquad \forall ij \in \Omega_L, e \in \Omega_{se}, t \in T \tag{78}$$

$$P_{ij,e,t}^{+} + P_{ij,e,t}^{-} = \sum_{y=1}^{Y} \Delta P_{ij,e,t,y} \quad \forall ij \in \Omega_L, e \in \Omega_{se}, t \in T \tag{79}$$

$$Q_{ij,e,t}^{+} + Q_{ij,e,t}^{-} = \sum_{y=1}^{Y} \Delta Q_{ij,e,t,y} \quad \forall ij \in \Omega_L, e \in \Omega_{se}, t \in T \tag{80}$$

$$0 \le \Delta P_{ij,e,t,y} \le \bar{\Delta}^L \quad \forall ij \in \Omega_L, e \in \Omega_{se}, t \in T, y \in Y \tag{81}$$

$$0 \le \Delta Q_{ij,e,t,y} \le \bar{\Delta}^L \quad \forall ij \in \Omega_L, e \in \Omega_{se}, t \in T, y \in Y \tag{82}$$

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$$\bar{\Delta}^L = \frac{\overline{V}^{Max}}{Y} max\{S_a^p, a \in \Omega_a\} \quad (83)$$

E. Linearization of Equation (41)

The nonlinearity in equation (41) occurs due to the product of two continuous variables. This can be easily linearized by using the binary expansion approach as in [38]. Since the BWIP ranges between $\underline{\gamma}$ and $\overline{\gamma}$ as in (51), the BWIP can be approximated discretely as follows:

$$\gamma_{dg,i} = \underline{\gamma} + \Delta\gamma \sum_{h=1}^{H+1} 2^{(h-1)} v_{h,dg,i} \qquad \forall dg \in \Omega_{DG}, i \in \Omega_N \quad (84)$$

where $v_{h,dg,i}$ is a binary variable, $\Delta\gamma = \frac{\overline{\gamma}-\underline{\gamma}}{W}$, and $W = 2^H$ for some non-negative integer value *H*. By multiplying both sides with $P_{DG_{dg,i,t}}$, equation (84) can be rewritten as follows:

$$INC_{dg,i,t} = \underline{\gamma} P_{DG_{dg,i,t}} + \Delta\gamma \sum_{h=1}^{H+1} 2^{(h-1)} d_{h,dg,i,t} \quad \forall d \in \Omega_{DG}, i \in \Omega_N, t \in T \quad (85)$$

where $d_{h,dg,i,t} = v_{h,dg,i} P_{DG_{dg,i,t}}$. The bilinear product can be transformed into a linear expression using the big-M approach as follows:

$$0 \le P_{DG_{dg,i,t}} - d_{h,dg,i,t} \le M(1 - v_{h,dg,i}) \quad \forall dg \in \Omega_{DG}, i \in \Omega_N, t \in T, h = 1,2,..,H+1 \quad (86)$$

$$0 \le d_{h,dg,i,t} \le M v_{h,dg,i} \quad \forall dg \in \Omega_{DG}, i \in \Omega_N, t \in T, h = 1,2,..,H+1 \quad (87)$$

F. MILP Model for the Proposed IDSEP

The MINLP formulation of the proposed IDSEP model is transformed to MILP considering the linearization techniques applied in section IV. Therefore, the full MILP model for the proposed IDSEP model is defined as follows

IDSEP Model	
Objective:	*Min* (13)
Constraints:	(14)-(21), (26)-(27), (29)-(30), (33)-(40), (42)-(46), (48)-(50),(52)-(56), (59)-(63), (66)-(83), and (85)-(87)

V. Numerical Results

A. System Under Study

The proposed IDSEP model was tested using a primary 54-node distribution system, whose full data can be found in [39]. The planning horizon is assumed to be 15 years with 3 % annual load growth. The planning horizon is divided into three stages, each of which has a five-year period (*K*). The data of expansion alternatives for substations and feeders can be found in [40] and [41]. The cost of energy losses is 50 US$/MWh, and the substation operation cost is 1 $(\text{US\$}/(\text{MVA})^2\ \text{h}))$ [35]. The interest rate is assumed to be 12.5 %, and the system power factor is 0.9. The costs of purchasing power from the market corresponding to the off-peak, mid-peak, and on-peak load states are forecasted using 15 years of energy price data obtained from [42]. Table II shows the market energy prices for all time stages and demand states. Investment and operation costs for each DG type can be found in [13], [43], and [44]. The maximum DG capacity at each bus is equal to 10 MW, and the penetration level for renewable-based DGs (μ) is assumed to be 20%, with 10% for each type. Historical wind speed, solar irradiance, and system demand data were obtained from [42], [45], [46].

TABLE II
COSTS OF ENERGY PURCHASED FROM THE MARKET ($/MWh)

	Off-peak	Mid-peak	On-peak
Stage 1	37.85	49.13	55.9
Stage 2	33.67	42.6	47.7
Stage 3	29.5	36.38	40.1

B. Uncertainty Modeling Results

The historical data used in this paper are analyzed based on the procedures described in section II. The results revealed that the Normal distribution was found to be the best distribution for mimicking fluctuations in system demand, while the Weibull distribution was best fit for modeling the wind speed variations. The Beta distribution is the best fit to model the solar irradiances. The parameters of the selected PDFs are listed in Table III.

TABLE III
BEST FITTING PROBABILITY DISTRIBUTION RESULTS

	Best Fitted PDF	Distribution Parameters
System demand (p.u.)	Normal	Mean = 0.69, Stdev. = 0.1
Wind speed (m/s)	Weibull	Shape =1.9, Scale = 6.07
Solar irradiance (kW/m^2)	Beta	Alpha = 0.27, Beta =1.3

C. Case Studies and Results

To validate the proposed IDSEP model, two case studies were conducted: 1) IDSEP with controllable DGs (CDG), and 2) IDSEP with controllable, wind, and PV-based DGs. For the work described in this section, the proposed IDSEP was designed based on the IRR of the DG investments only, and the MARR for each DG type was assumed to be 10 %. The results of these case studies are summarized in Tables IV, V, and VI. Table IV presents the net present values (NPV) of the planning costs incurred by the LDC, with a breakdown of costs for each case. Table V shows the NPV of the DG project benefits and the optimal BWIP price that guarantees the financial feasibility of each DG investment at each bus. Table VI lists the planning decisions committed to by the LDC and DG investors.

TABLE IV
NPV FOR PLANNING COSTS TO BE INCURRED BY THE LDC, IN (10^6US$)

	Base case	Case 1	Case 2
Substation investment (2)	22.0	0.00	2.46
Substation operation (4)	1.83	0.52	0.63
Feeder investment (3)	4.76	1.36	1.71
Cost of energy losses (5)	0.82	0.35	0.41
Cost of energy purchased from the market (6)	90.5	40.11	45.12
Cost of energy purchased from CDG (7)	0.00	54.02	47.31
Cost of energy purchased from WDG (7)	0.00	0.00	2.27
Cost of energy purchased from PVDG (7)	0.00	0.00	2.92
Total NPV of planning costs	119.9	96.3	102.8
NPV of the net savings for LDC	0.00	23.6	17.1

1) IDSEP with Controllable DGs (CDG): In this case, which deals only with controllable DGs, the results revealed that the NPV of the planning costs incurred by the LDC is 96.3×10^6 US\$. Almost 41.6 % of these costs represent the cost of purchasing energy from the market, whereas 56 % of the costs represent the cost of purchasing energy generated by controllable DGs, as shown in Table IV. A comparison of these numbers with the base case results when DGs are not considered reveals that the savings the LDC can gain from inserting DGs is 77.62×10^6 US\$. However, the LDC should spend 54.02×10^6 US\$ as incentives for DG investors, making the net LDC savings 23.6×10^6 US\$. The DG investor plans are indicated in Table VI. Nine locations are identified as optimal for integrating the DGs, and the cumulative DG capacity at each location for each planning stage is shown in Table VI. Table V displays the BWIP long-term contract price committed to for each DG and the NPV for the DG benefits. The BWIP prices vary from 42.38 US\$/MWh to 43.45 US\$/MWh, depending on the capacity of each DG at each stage and the required MARR. These prices guarantee that the project is financially feasible at each bus where the IRRs equal 10 %. For this scenario, there was no need for either a substation upgrade or construction plans since the anticipated growth in energy consumption for each stage is met by the contracted DGs. The LDC must upgrade five feeders in stage 1, two feeders in stage 2, and one feeder in stage 3, as noted in Table VI.

TABLE V
OPTIMAL DG BWIP PRICES AND INCOMES

		Bus No.	BWIP price (\$/MWh)	NPV of DG income (Benefit) (10^6US\$)
Case 1	**CDG**	4	42.38	4.67
		8	42.38	8.40
		13	42.38	5.23
		19	42.38	0.56
		24	43.45	12.78
		32	42.5	7.76
		38	42.6	10.1
		41	42.6	6.48
		50	43.4	4.82
Case 2	**CDG**	1	45.32	0.94
		6	42.38	9.01
		10	42.38	10.8
		16	42.45	5.41
		25	45	2.07
		30	45	1.45
		36	42.7	13.76
		42	42.38	7.78
		50	42.7	3.42
	WDG	12	109.5	0.37
		18	110.3	1.54
		23	111.7	0.41
		33	86	0.44
	PVDG	3	124.3	0.554
		9	123.8	1.22
		20	125.75	1.73

All of the feeder upgrade plans utilized feeder alternative A1. An interesting finding is that the average incentive price is equal to 42.67 \$/MWh, higher than the average price of purchasing energy from the market, which would cost 40 \$/MWh. However, it is more economical for the LDC to purchase from the DG owners at this price since the presence of the DGs enables the deferment of most of the feeder upgrade decisions, reduces the cost of energy losses, and eliminates the need for substation upgrade decisions. In other words, the incremental cost of purchasing energy from CDGs at the incremental price (*incentive price-market price*) is lower than the cost of upgrading the substations and the feeders. Fig. 2 illustrates the network topology for case 1.

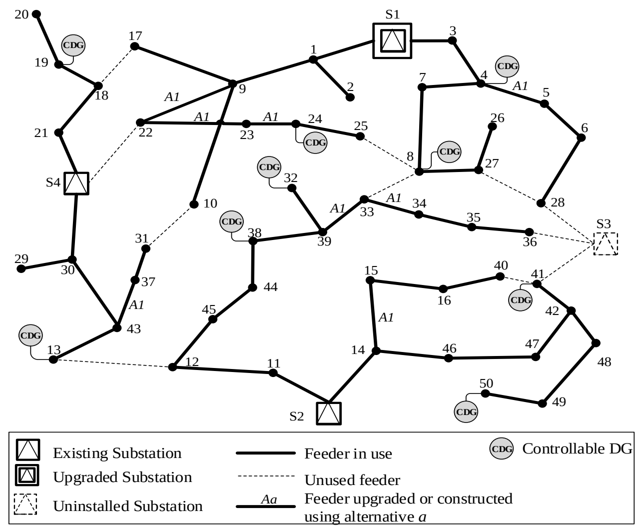

Fig. 2. Network topology for case 1 with investments.

2) IDSEP with CDGs, WDGs, and PVDGs: The NPV of the total planning costs in the case in which all DG types are included in the model is 102.8×10^6 US\$. As can be seen, the LDC can save almost 69.6×10^6 US\$ by introducing these DGs into the grid. However, the LDC must spend 47.31×10^6 US\$, 2.27×10^6 US\$, and 2.92×10^6 US\$ to incentivize CDG, WDG, and PVDG owners, respectively, with the incentives being distributed so as to ensure the feasibility of the DG projects. The total net savings with this scenario are therefore 17.1×10^6 US\$. DG investments are located at a total of 16 system buses, as evident in stage 3. The penetration level of renewable DGs is 20 %, 10% for each renewable-based DG. Since the IRRs equal the 10 %, as determined by the investors, the contracted BWIP prices shown in Table V guarantee that the DG projects are financially feasible at all of the defined buses for all DG types. The WDG contract price at bus 33 is 86 US\$/MWh, which is lower than that at buses 12, 18, and 23. This price is acceptable since the investment in bus 33 is required at stage 2 for a short contract period (ten years), and it has lower NPV of installation cost. Fig. 3 illustrates the network topology for case 2. The planned network topology in this case remains the same as in case 1; however, the locations of DG integration are changed as shown in Fig. 2 and Fig. 3. It can be observed that most LDC investment plans are deferred and that the feeder-upgrade investment costs in this case are slightly higher than the costs obtained in case 1 due to the need for higher feeder capacities. A substation upgrade decision was produced for substation 1 at the third stage using the substation upgrade alternative 1.

D. Incentive Design Based on the Profitability Index

The previous section (Section V-C) dealt with an IDSEP design based on the specified MARR of the DG investors. However, it is more appropriate and convenient for DG investors to apply other economic measures to ensure the profitability of their projects. This section discusses an IDSEP design based on the PI, addressing the results for both case 1 and case 2. For case 1,

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in which only CDGs are considered, Fig. 4 shows the variations in the NPV of the LDC costs and total incentive costs, along with the changes in the PI. As long as the PI increases, the NPV of the LDC costs increases, and the LDC savings decrease. It can also be seen that when the PI reaches 1.53, LDC costs are almost equal to the base case cost for LDC expansion plans with no DGs, and consequently the net savings are equal to zero. The LDC should therefore avoid designing the system with a PI above 1.53.

TABLE VI
INVESTMENT PLANS COMMITTED TO BY THE LDC AND DG INVESTORS FOR EACH STAGE

	Case 1		Case 2			
Stage	**LDC Plans**	**CDG Owner Plans**	**LDC Plans**	**CDG Owner Plans**	**WDG Owner Plans**	**PVDG Owner Plans**
1	14-15 (A1) 22-23 (A1) 23-24 (A1) 33-39 (A1) 37-43 (A1)	4 (1.65) 8 (2.97) 13 (1.85) 19 (0.2) 24 (3.72) 32 (2.67) 38 (3.44) 41 (2.23) 50 (1.55)	30-43 (A2) 33-34 (A2) 34-35 (A2) 35-36 (A2) 37-43 (A1)	6 (3.2) 10 (3.8) 16 (1.87) 25 (0.15) 30 (0.05) 36 (4.46) 42 (2.75) 50 (1.2)	12 (0.1) 18 (0.3) 23 (0.1)	3 (0.17) 9 (0.33) 20 (0.5)
2	4-5 (A1) 33-34 (A1)	4 (1.65) 8 (2.97) 13 (1.85) 19 (0.2) 24 (4.54) 32 (2.77) 38 (3.54) 41 (2.23) 50 (1.55)		1 (0.62) 6 (3.2) 10 (3.8) 16 (1.95) 25 (1.18) 30 (0.92) 36 (5.03) 42 (2.75) 50 (1.2)	12 (0.1) 18 (0.3) 23 (0.1) 33 (0.5)	3 (0.17) 9 (0.33) 20 (0.5)
3	9-22 (A1)	4 (1.65) 8 (2.97) 13 (1.85) 19 (0.2) 24 (6) 32 (2.87) 38 (3.8) 41 (2.47) 50 (2.22)	S1 (U1) S1-1 (A3) S1-3 (A3) S4-30 (A3) 18-19 (A1) 18-21 (A2)	1 (0.62) 6 (3.2) 10 (3.8) 16 (1.95) 25 (1.3) 30 (0.92) 36 (5.46) 42 (2.75) 50 (1.2)	12 (0.6) 18 (2.6) 23 (0.7) 33 (0.6)	3 (0.7) 9 (1.6) 20 (2.3)

For LDC plans, (U) represents a substation upgrade alternative, (C) represents a substation construction alternative, and (A) represents a feeder alternative. For DG investor plans, the first number represents the bus number and the number in parentheses represents the cumulative DG capacity in MW.

It can be observed that although the incentive prices are slightly higher than the average purchasing price from the market, the proposed model found that it is more economical for the LDC to form contracts with the DG investors since the defined locations and capacities of the DGs will eliminate the upgrade investments of the substations, reduce the line investments, and minimize the losses and operation costs. The average prices for the BWIP and the average DPP for CDG projects can be seen in Fig. 5.

For case 2, in which all types of DGs are considered, the results also reveal that when the PI increases, the BWIP prices and the total LDC costs increase as well, as shown in Fig. 6 and Fig. 7. From another perspective, as long as the PI increases, the net LDC savings decrease until a threshold point is reached, which is almost 1.4, the point at which the LDC cost is equal to the base case cost. The LDC should therefore not design the system with a PI above 1.4. It should be noted that the incentive costs for WDGs and PVDGs increase along with the rising PI. This increase would be expected regardless of a BWIP price that is higher than the average market price in order to satisfy the constraint imposed on renewable-based DG penetration.

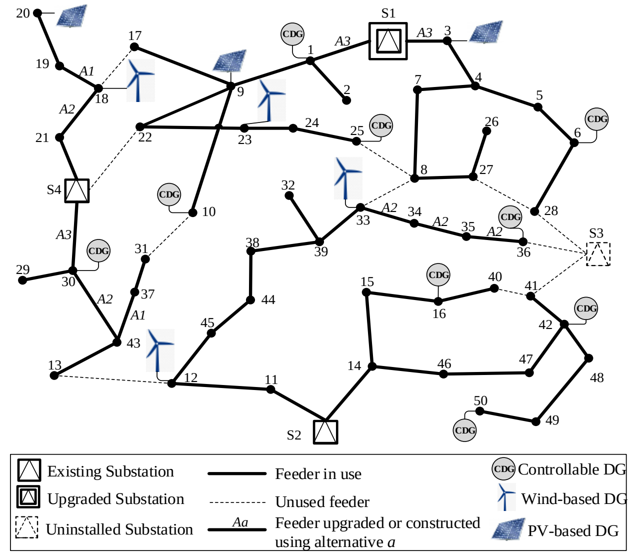

Fig. 3. Network topology for case 2 with investments.

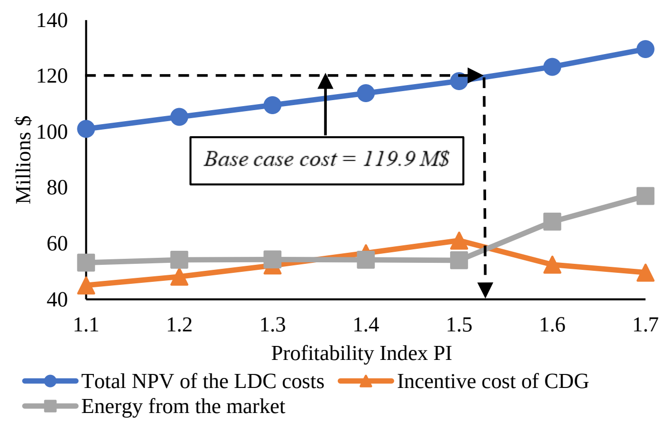

Fig. 4. Variations in planning costs with different PIs for case 1 (CDGs only).

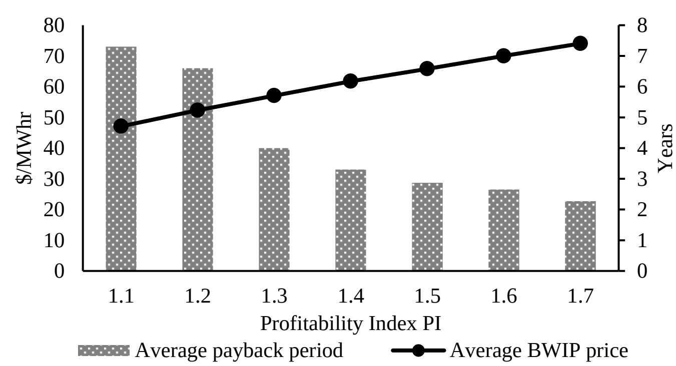

Fig. 5. Variations in the average BWIP prices and the DPP with different PIs for case 1 (CDGs only).

As expected, although the average BWIP price for CDGs is higher than the average market price, it is still more economical for the LDC to purchase power at that price to avoid or defer substation upgrade costs, as indicated in Fig. 6. The average BWIP price for each DG type and the average payback period are shown in Fig. 7.

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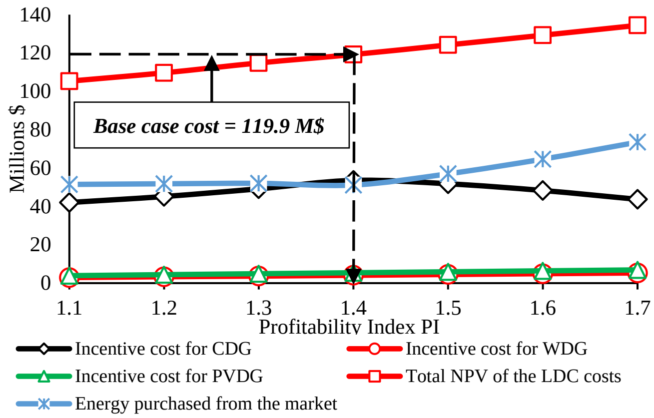

Fig. 6. Variations in planning costs with different PIs for case 2.

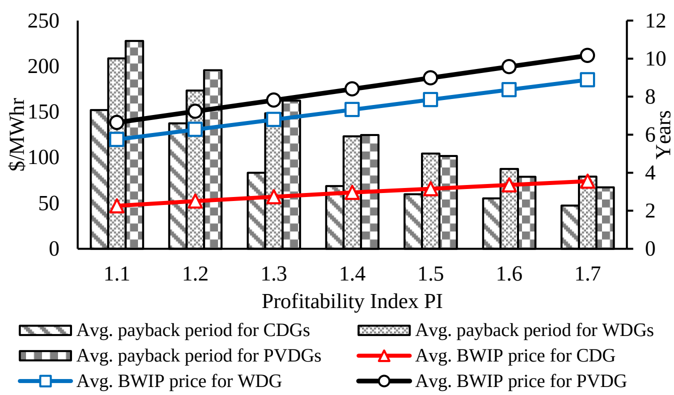

Fig. 7. Variations in average BWIP prices and DPPs with different PIs for case 2.

E. Effect of Uncertainty on Planning Results

To examine the results of the proposed model from the uncertainty perspective (i.e. uncertainty of system demand, wind and PV-based DG output power, and energy prices), a Monte Carlo Simulation (MCS) coupled with power flow analysis [47] has been executed for a large number of iterations (i.e. 10,000 iterations).

1) Planning costs and profitability indices

The effect of uncertainty upon planning costs and profitability indices is studied in this section. It can be observed that, at different profitability indices, the total planning costs obtained from the proposed model are very close to those obtained using MCS. Moreover, the differences between the designed PIs and the evaluated PIs using MCS are very small, as can be seen in Table VII. These results provide evidence that the uncertainty model captures the system randomness efficiently.

TABLE VII
COMPARISON BETWEEN THE PROPOSED MODEL AND MCS RESULTS

Proposed Model Results		MCS Results	
PI	**Total Cost (M$)**	**PI**	**Total Cost (M$)**
1.1	105.30	1.115	105.62
1.2	109.65	1.197	109.325
1.3	114.80	1.295	114.24
1.4	119.21	1.403	120.02
1.5	124.20	1.502	124.68
1.6	129.25	1.599	128.98
1.7	134.40	1.702	134.83

2) Planned network topology robustness

The robustness of the network planned topology can be assessed through the use of MCS-based probabilistic power flow. With a 95% confidence level, it can be observed in Fig. 8 that the voltages at each bus in the system are within the permissible limit (i.e. 0.95-1.05 p.u.). Moreover, with a 95% confidence level, in can be observed in Fig. 9 that the feeder currents are within the designed thermal capacities of the lines taking into account the new capacities of the upgraded feeders obtained from the model outcomes. These two assessments provide a very good indication that the planned topology is robust with respect to the uncertainty caused by the fluctuations of system demand and renewable-based DGs output power.

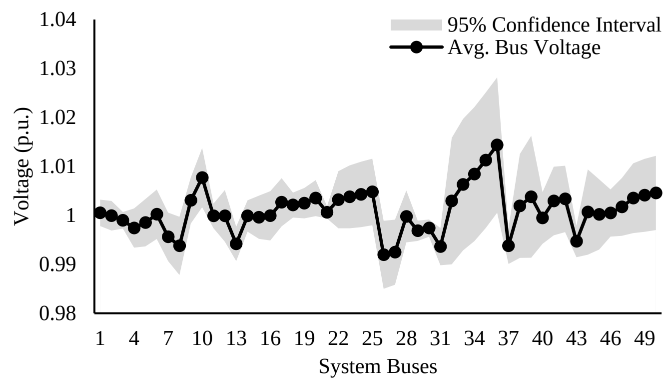

Fig. 8. Avg. system buses voltages and their 95% confidence intervals.

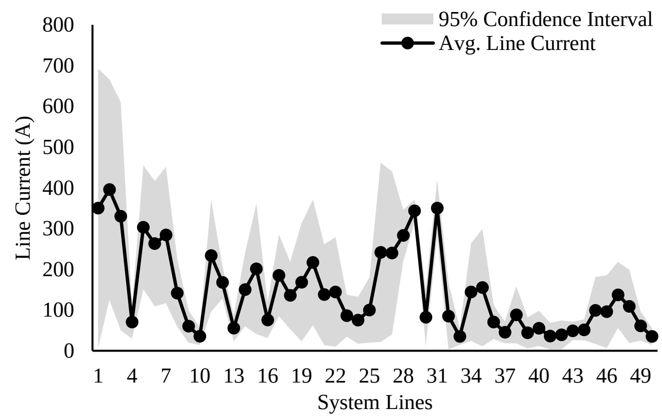

Fig. 9. Avg. system lines currents and their 95% confidence intervals.

F. Comparing Multistage and Single Stage Models

The proposed IDSEP model is a dynamic model (i.e. multistage-based model) in which the planning decisions take place at different time stages in the planning horizon based on the system needs, following the load growth at each stage. Thus, to present the advantages of the multistage model over a single stage model, the planning model is solved using a single stage (i.e. a 15-year planning period) where the planning investments occur at the beginning of the planning period (i.e. year 1) considering the demand in the last stage. The single stage results showed that the total planning cost for case 1 and case 2 are 102.27 and 108.46× 10^6 US$, respectively. These results are higher than the multistage results obtained by the proposed model. The multistage model allows for efficient utilization of the investments over the entire planning period.

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G. Computational Aspects

The mixed integer linear programming (MILP) optimization model was solved by utilizing the CPLEX solver with programming and execution in GAMS environment [48] using a desktop computer with an Intel® Core™ i7 3.60 GHz processor and 16 GB of RAM. CPLEX solver utilizes Branch and Cut-based algorithm to solve the proposed model with an optimality gap set to 1%. For Case 1 with only CDG, the elapsed time is 12.3 minutes, and for Case 2 with all DG types, the solver takes 722 minutes to reach the optimal solution. Considering that the planning studies are basically offline problems, the computational effort is not a primary concern. This, combined with the fact that the equations and the variables of the proposed model can accommodate any increase in the system size without causing model breakdown, the proposed model is applicable for large scale distribution systems.

VI. Conclusion

This paper has presented a novel IDSEP model that incorporates the active participation of DG investors in the expansion problem. The proposed model establishes a BWIP and determines the incentives that should be offered by the LDC to DG investors. The proposed model enables the LDC to direct the connection of DG projects to specific buses that will benefit the overall system and that will ensure the profitability of the investments of the corresponding investors based on the BWIP prices offered. The IDSEP model takes into account DG installation and operation by the investor and analyzes several economic indices: the MARR, PI, and DPP of the DG projects. At the same time, the LDC has the opportunity to identify the least cost solution from a combination of the proposed BWIP and traditional expansion options. In this way the model allows the LDC to coordinate its future expansion projects effectively with DG investors. Three types of DGs are considered: controllable, wind-based, and PV-based. The uncertainty associated with the intermittent nature of wind speed, solar irradiance, and system demand is treated probabilistically, and all possible operating scenarios are created. A number of linearization methods are used to convert the MINLP model into a MILP model. The results of the case studies presented demonstrate the effectiveness of the proposed model, which will encourage DG investors to play a crucial role in the distribution expansion process, increase LDC savings, guarantee the profitability of DG projects, and consequently minimize total planning costs.

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